Physics I
ISI B.Math
Final Exam : May 9 , 2018

Total Marks: 50

Answer all questions

1.(**Marks** $= \mathbf{2+4+2+2=10}$)

Consider the motion a particle of mass m under the influence of a force $\mathbf{F} = -k\mathbf{r}$, where k is a positive constant.

(a) Show that the motion of the particle lies in a plane

(b) Find the position of the particle as a function of time given that at $t = 0, x = a, y = 0; v_x = 0. v_y = v, z = 0, v_z = 0$ and show that the orbit is an ellipse.

(c) Find the time period of the orbit

(d) Does the particle obey Kepler's Laws , assuming the sun is at the origin exerting the above force ? Explain.

2. (**Marks** $= \mathbf{3+1+3+3=10}$)

Consider a particle of mass m constrained to move on the surface of a cylinder defined by $x^2+y^2 = R^2$. The particle is subject to a force directed towards the origin and proportional to the distance of the particle from the origin $\mathbf{F} = -k\mathbf{r}$ where k is a positive constant.

(a) Write down the Lagrangian of the system in terms of the generalized coordinates z and θ, where z, θ are the standard cylindrical coordinates. Is the Hamiltonian of the system conserved ?

(b) Identify the cyclic coordinate.

(c) From the Lagrangian, find the equations of motion for z and θ. Show that the generalized momentum corresponding to the cyclic coordinate is conserved.

(d) If at $t = 0$, the particle is at $z = 0$, with $\dot{z} = v$ and the initial angular momentum is L , find $z(t)$ and $\theta(t)$.

3. (**Marks** $= \mathbf{5+5=10}$)

(a) Two particles with masses m_1, m_2 and velocities $\mathbf{v_1}, \mathbf{v_2}$ collide and stick together. Show that the loss in kinetic energy due to collision is

$$\frac{m_1 m_2}{2(m_1+m_2)}|\mathbf{v_1}-\mathbf{v_2}|^2$$

(b) A body of mass $4m$ is at rest when it explodes into three fragments of masses $2m$, m and m. After the explosion, the two fragments of mass m are observed to be moving with equal speed in

directions making 120° with each other. What is the velocity of the centre of mass after the collision ? Find the proportion of the total kinetic energy carried by each fragment.

4. (**Marks** $= \mathbf{2 + 5 + 3 = 10}$)

(a) Find the number of degrees of freedom of i) a simple pendulum moving in a vertical plane ii) two masses connected by a rigid rod iii) a rigid rod sliding on a flat table iv) A marble rolling inside a hemispherical bowl.

(b) Find the inertia tensor of a homogeneous cube of density ρ, mass M and side a when one corner is at the origin and the three adjacent edges lie along the positive x_1, x_2,x_3 coordinate axes. Recall that the form of the inertia tensor is given by $I_{ij} = \int_V \rho(\mathbf{r}) \left(\delta_{ij} \sum_k {x_k}^2 - x_i x_j\right) dv$ where $dv = dx_1 dx_2 dx_3$ is the element of volume at the position defined by the vector $\mathbf{r}$, and where V is the volume of the body.

(c) In the example of part (b), do the principal axes of inertia lie along the coordinate axes ? Explain. If the above cube rotates about the x_3 axis with an angular velocity $\omega = \omega \mathbf{e_3}$, where $\mathbf{e_3}$ is a unit vector in the x_3 direction, find the component L_3 of the angular momentum along the same direction. Will the angular momentum vector point in the same direction ?

5.(**Marks** $= \mathbf{4 + 6 = 10}$)

(a) State whether the following statements are true or false with a very brief (one or two lines) explanation.
i) A particle moving under the potential $U(r) = Kr^5$ where K is a positive constant can have a circular orbit
ii) A point mass m travels in a circle of radius R with centre at the point $(0, 0, z_0)$ with the plane of the circle parallel to the $x - y$ plane with an angular velocity $\omega \hat{\mathbf{z}}$.The angular momentum vector about the origin points in the same direction as the angular velocity.
iii) A particle of mass m travelling with velocity $\mathbf{v}$ collides with a stationary particle of mass M. In the centre of mass frame both particles will move with equal and opposite velocities before and after the collision.
iv) The point of support of a simple pendulum oscillates according to the equation $x = a \cos \omega t$. The total energy of the pendulum is not conserved.

(b) A mass m travels perpendicular to a stick of mass m and length l, which is initially at rest. After the collision, the center of the stick and mass move with equal velocities. Find the angular speed of the stick as it rotates about the centre after the collision. At what distance from the centre of the stick does the collision with the mass take place ?